Radial Migration of Spherical Particles in Couette Systems

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Several studies have been made in Couette systems of the primary motion (angular translation and rotation) of spheres suspended in viscous liquids. Radial migration trajectories, however, have been unreported for single spheres.

In this study, a prediction of radial migration is developed and compared with experimental measurements. The predicted trajectories were found to agree well with the measured ones.

Ultimately, radially migrating spheres reached an equilibrium position located approximately midway between the cylinder walls. The model developed predicted an equilibrium position just slightly inside the midpoint, while measured equilibrium positions fell between the midpoint and 0.4, the annular thickness from the inner cylinder walls. The differences in equilibrium positions are attributed to the approximate nature of the model.

Segré and Silberberg's (*1*) observation of radial migration of spheres suspended in a Newtonian liquid flowing through a tube stimulated considerable interest in such migration phenomena. Most subsequent experimental and theoretical works have been restricted to tube systems. Although Poiseuille flow systems have been given considerable attention, a basically simpler system, the Couette system, has been essentially ignored. Because the shear induced in a properly proportioned Couette system is nearly constant, it offers theoretical problems which are more simply approached than those posed by the Poiseuille system. The present work derives a semiempirical lateral force for use in predicting radial migration in a Couette system, a system more easily analyzed than the tube system, and hence more adapted to establishing the fundamental reasons for migration phenomena.

Segré and Silberberg's (*1*) initial observations with neutrally buoyant spheres, freely translating and rotating, convincingly demonstrated the existence of laterally directed forces. Extensions of their observations include those to nonrotating spheres (*2*) and to spheres with a density different from the density of the liquid (*3*). Several other authors have reported on various aspects of this phenomena (*4, 5*).

Several authors have derived expressions for the lateral force acting on a sphere under viscous flow conditions. Rubinow and Keller (*6*) derived such an expression by considering a rotating sphere translating through a stationary Newtonian fluid and applying the method of matched asymptotic expansions.

A laterally directed force proportional to the velocity of translation of the sphere and its rate of rotation was derived. This result has been applied to explain migration phenomena in Poiseuille systems even though the result applies only to a quiescent fluid. While limited success has been obtained with this result in explaining experimental Poiseuille data, there are fundamental objections to such applications (*7*).

Saffman (*8*) calculated a laterally directed surface force by considering a rotating sphere translating through a sheared fluid. The direction of translation was parallel to the free stream velocity. Saffman concluded that the laterally directed force was proportional to the translational lag velocity (termed a lag velocity because it is the translational velocity of the sphere relative to that of the unperturbed fluid). In this first-order approximation, it was found that the force was independent of the sphere's rotation.

Recently, Harper and Chang (*9*) have found Saffman's result to be too large by a factor of 4π. They have also calculated the lift force resulting from the sphere's translation perpendicular to the free stream velocity.

A preliminary report of sphere trajectories in a Couette system has been presented by Halow and Wills (*10*). Here, there is developed a semiempirical model for the radial forces in a Couette system, this model containing a single parameter to be determined experimentally. Also, an approximate expression is developed for the sphere's lag velocity. These unique results are combined with the results of others to develop a complete model for trajectory analysis in Couette systems. Trajectories predicted from the model compare favorably with those measured experimentally.

FORMULATION OF THE COUETTE SPHERE PROBLEM

A spherical particle suspended in a homogeneous Newtonian fluid will be considered. The fluid and the sphere are contained in an annular region bounded by two concentric cylinders, and a shear is induced in the fluid by rotation of the inner cylinder about its axis. The only forces acting on such a sphere are the surface forces exerted by the fluid and body forces arising from a density difference between the sphere and the fluid. The Couette system will be oriented with its axis parallel to the direction of the gravitational force. The body forces then consist of a gravitational force parallel to the axis of the Couette system and a centripetal force perpendicular to this axis. The sphere is free to translate in the annular region and to rotate about its own axis. Its translational movement is governed by Newton's second law of motion which can be written as

$$m_s \frac{d\mathbf{v}}{dt} = \mathbf{F}_s + \mathbf{F}_b \tag{1}$$

The body forces are readily expressed in terms of the velocity and position of the sphere together with several parameters characteristic of the sphere and the liquid. If an expression for the surface forces and the expressions for the body forces are substituted into Equation (1), Newton's second law of motion, written in terms of the radial and vertical position of the sphere and its tangential velocity, is

$$m_s \frac{d^2 r_s}{dt^2} = \left[\int_s \int \mathfrak{S} \cdot \mathbf{n}\, ds \right] \cdot \mathbf{e}_r + \left(\frac{1}{6} \pi D_s^3 \right) (\rho_s - \rho_l) \left(\frac{v_t^2}{r_s} \right) \quad (2)$$

$$m_s \frac{dv_t}{dt} = \left[\int_s \int \mathfrak{S} \cdot \mathbf{n}\, ds \right] \cdot \mathbf{e}_t \quad (3)$$

$$m_s \frac{d^2 z_s}{dt^2} = \left[\int_s \int \mathfrak{S} \cdot \mathbf{n}\, ds \right] \cdot \mathbf{e}_z - \left(\frac{1}{6} \pi D_s^3 \right) (\rho_s - \rho_l) g \quad (4)$$

The initial conditions associated with these equations are specification of the sphere's initial radial and vertical positions and the sphere's initial velocity.

The appearance of the stress tensor in Equations (2) to (4) couples these equations to the velocity and pressure fields associated with the liquid. This coupling occurs through the definition of the stress tensor for a Newtonian fluid in terms of the velocity and pressure fields of the fluid. A complete mathematical formulation of the problem requires additional relations between these fields, namely, a specialized form of the Navier-Stokes equations. These equations have associated boundary conditions which couple them to the Newton's second law equations for translation and also the analogous equations for the rotation of the sphere. It is, however, unnecessary to consider explicitly such a complete formulation to calculate sphere trajectories.*

If the restriction of incompressibility is now imposed on the fluid, all that is required is recognition of the role of the applicable specialization of the Navier-Stokes equations and the following boundary conditions associated with these equations:

$$\text{at } r = R_i, \mathbf{u} = R_i \Omega_i \mathbf{e}_t \quad \text{(inner cylinder)} \quad (5)$$

$$\text{at } r = R_o, \mathbf{u} = 0 \quad \text{(outer cylinder)} \quad (6)$$

$$\text{at } r_s^1 = D_s/2, \mathbf{u} = \mathbf{v} + \boldsymbol{\omega}_s \quad \text{(sphere)} \quad (7)$$

If a linearization of the specialized form of the Navier-Stokes equations is performed, then surface forces can be approximated by superimposing the forces developed in a number of simpler problems. If the superimposed velocity-pressure field pairs are chosen so that the composite fields approximately satisfy the boundary conditions associated with the fluid in the complete formulation, then an approximate solution of the complete Couette sphere problem can be obtained.

THE COMPONENT OF THE SURFACE FORCE CAUSING RADIAL MIGRATION

A lift force similar to that derived by Saffman for a sphere immersed in an infinite fluid with a simple shear is taken as the first of the two radial components of the surface force. We note that Harper (9) has found Saffman's lift to be in error by a factor of $1/4\pi$. The trajectory data (to be presented later) indicate that a correction to Saffman's force of only 1/2.5 is required to give good agreement between measured and calculated trajectories. An empirical multiplier of 5.0 is then present in our lift force, which is otherwise identical to the corrected Saffman force.

A velocity difference between the center of the sphere and that of the fluid at infinity along a line through the center of the sphere and parallel to the direction of flow is assumed. The origin of the rectangular Cartesian coordinate system has been taken at the center of the sphere. Figure 1 depicts this arrangement. The free stream velocity is

$$\mathbf{u}_{fs} = (w + \kappa x)\, \mathbf{e}_y \quad (8)$$

The parameter κ in Equation (8) can be readily identified with the Couette parameters as

$$\kappa = \frac{\Omega_i R_i}{R_o - R_i} \quad (9)$$

This identification utilizes the approximately linear nature of the Couette velocity field. The lag velocity can similarly be identified as

$$w = u_t - v_t \quad (10)$$

The lift force will be given the symbol $\mathbf{f}^{(1)}$, and the result of Saffman's analysis with Chang's correction and our empirical constant can be written in the present notation as

$$\mathbf{f}^{(1)} = -8.09\, D_s^2 (\mu \Omega_i \rho_l R_i)^{1/2} (R_o - R_i)^{-1/2} w\, \mathbf{e}_r \quad (11)$$

Note that this equation states that the lift force is directly proportional to the lag velocity of the sphere. If a result such as this is applicable to a neutrally buoyant sphere moving radially in a Couette system, then the lag velocity must change sign on either side of the equilibrium position located about midway between the walls. For this force to have the correct sign, the sphere must lead the liquid when it is on the outer side of this position. This change in sign of the lag velocity is taken to be the fundamental reason for the observed radial migration phenomena in Couette systems.

THE RADIAL DRAG FORCE

A second radial force, one retarding migration, is found by considering the drag exerted on a sphere moving in an infinite stationary fluid and adding to this drag a correction for each of two infinite plane walls. In this way, the effect of the walls on the radial drag is built up from the known effect of a single wall on the drag exerted on a sphere moving perpendicular to it. This type of correction was first suggested by Oseen (12). Exceptions can be made to this technique of calculation, and this point will be discussed later. However, this method offers an analytic simplicity lacking in other types of corrections, such as those by Faxen (16), and it is chosen for the present analysis for this reason.

The symbol $\mathbf{f}^{(2)}$ will be used to represent the radial drag. The drag exerted on a sphere moving through an infinite stationary fluid is merely Stokes' law. Each of the plane walls contributes a correction of the Lorentz type to this drag inversely proportional to the distance between the center of the sphere and the wall. The radial drag is then

$$\mathbf{f}^{(2)} = -3\, \pi \mu D_s (v_r) \left\{ 1 + \frac{9\, D_s}{16} \left[\frac{1}{(r_s - R_i)} + \frac{1}{(R_o - r_s)} \right] \right\} \mathbf{e}_r \quad (12)$$

This equation is a first-order approximation to the true radial drag on the sphere. Each correction to Stokes' law results from developing a velocity field and a pressure field which will cancel the velocity field associated with a sphere moving through an infinite stationary fluid at each plane wall. The mirror image technique of Lorentz (13) was originally used to develop these corrections. Higher-order approximations could be obtained by successively applying the general solution of the Stokes equations for spherical geometries given by Lamb (14) and the mirror image technique. It is, however, unnecessary

* For a complete formulation of the problem including end effects and a dimensional analysis of the complete problem, see (11).

to do this in the present analysis.

THE TANGENTIAL DRAG FORCE

The drag exerted on the sphere in the tangential direction is developed next. The importance of this term lies in that it gives rise to the change of sign of the lag velocity required for migration phenomena. Application of the Lorentz (*13*) mirror image technique to the appropriate unbounded solution generates two wall corrections having opposite signs, one for each of the two walls. This difference in sign is taken to be the basic reason for prediction of radial migration phenomena in Couette systems.

Consider first the problem of a sphere immersed in an infinite fluid which contains a simple shear. This geometry, which is the same as the geometry considered by Saffman, has the sphere translating in the direction of bulk flow of the fluid. The drag exerted on such a sphere is

$$\mathbf{f}_1^{(3)} = 3\,\pi\mu\, D_s w\, \mathbf{e}_t \tag{13}$$

To calculate the correction to the tangential drag due to the inner wall, it is only necessary to know the value of the x component of the reflected velocity at the center of the sphere. This result is readily obtained by application of the mirror technique and evaluating the x component of the resulting velocity field $\mathbf{q}_1$ at the center of the sphere. The result of this computation is

$$[\mathbf{q}_1 \cdot \mathbf{e}_x] = \frac{9w}{32}\left(\frac{\tau}{R^*}\right) + \frac{5\kappa D_s}{128}\left(\frac{\tau}{R^*}\right)^2 \tag{14}$$

It is convenient to introduce the dimensionless variables T^*, R^*, V_t^*, τ, L^*, ρ^*, and N_{Re} into the analysis at this point. A dimensionless radial position R^* is defined as

$$R^* = \frac{(r - R_i)}{(R_o - R_i)} \tag{15}$$

This variable has the property of being zero at the inner cylinder wall and unity at the outer cylinder wall. The dimensionless variables $T^* = t\Omega_i$ and V_t^* result from the choice of R_i as the characteristic length and $1/\Omega_i$ as the characteristic time. The dimensionless groups τ, L^*, N_{Re}, and ρ^* rise during the reduction to dimensionless form.

The reflection of the $\mathbf{q}_2$ field from the outer cylinder wall could be accomplished in a manner quite analogous to that for the inner cylinder wall. However, it is simpler to geometrically transform the above result so that it applies to the outer wall reflection problem. Consideration of the geometry of the Couette system leads to the replacement of κ by $-\kappa$ and τ/R^* by $\tau/(1 - R^*)$ in Equation (14) to obtain the desired result. These changes give

$$[\mathbf{q}_2 \cdot \mathbf{e}_x] = \left[\frac{9w}{32}\right]\left[\frac{\tau}{(1 - R^*)}\right] - \left[\frac{5\kappa D_s}{128}\right]\left[\frac{\tau}{1 - R^*}\right]^2 \tag{16}$$

With the completion of the calculation of the two x components of the reflected fields, it is now possible to write the drag force in the tangential direction. This drag is found by replacing the lag velocity w in Equation (13) by w plus the two velocities given in Equations (14) and (16):

$$\mathbf{f}^{(3)} = 3\,\pi\mu D_s \left\{ w + \frac{9w}{32}\left[\frac{\tau}{R^*} + \frac{\tau}{1 - R^*}\right] + \frac{5\kappa D_s}{128}\left(\left[\frac{\tau}{R^*}\right]^2 - \left[\frac{\tau}{1 - R^*}\right]^2\right)\right\} \mathbf{e}_t \tag{17}$$

With the computation of the tangential surface force completed, the radial and tangential components of Newton's second law equations for the sphere's translation can be written independent of the vertical component of these equations and the Navier-Stokes equations. The solution of the radial and tangential component equations will lead to an approximate trajectory for the sphere.

SPHERE TRAJECTORY ANALYSIS

Newton's second law of motion can be written with

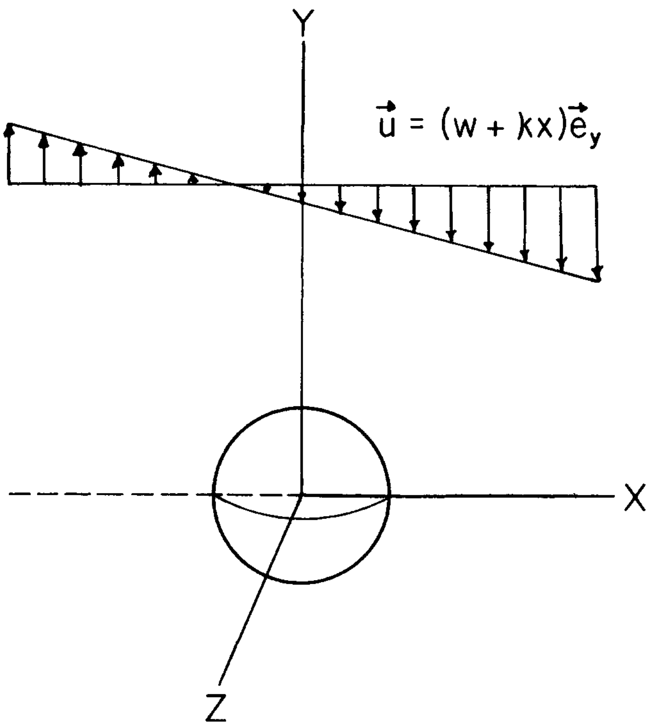

Fig. 1. A sphere in a fluid undergoing simple shear.

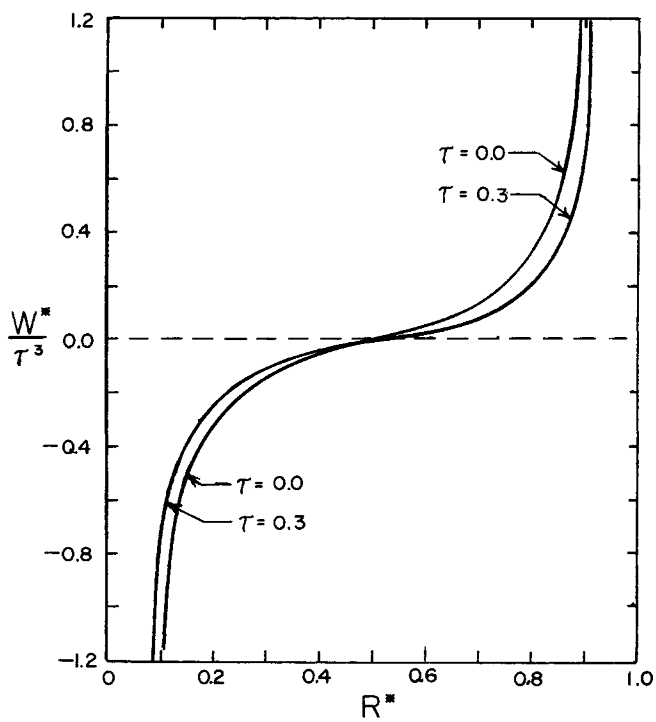

Fig. 2. Analytic solution: lag velocity vs. radial position.

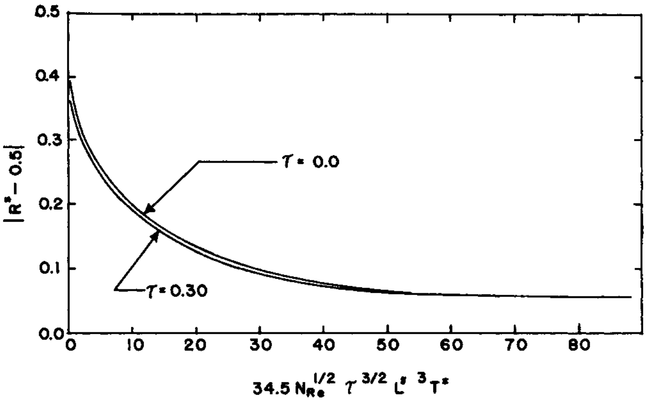

Fig. 3. Analytic solution: radial position vs. dimensionless scaled time.

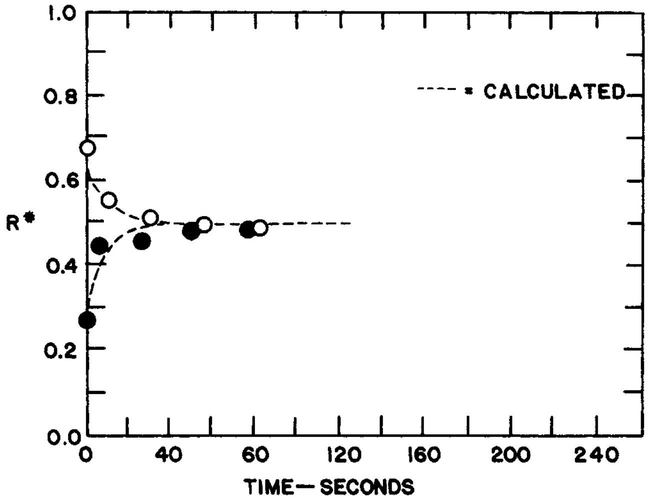

Fig. 4. Radial position vs. time: measured and calculated.

the dimensionless variables, by utilizing the approximate expressions for the forces, as

$$\frac{d^2R^*}{dT^{*2}} = -C_1W^* - C_2(1+2F^*)\frac{dR^*}{dT^*} - C_3H^* \quad (18)$$

$$\frac{dV_t^*}{dT^*} = C_2[W^*(1+F^*)+G^*] \quad (19)$$

where

$$C_1 = 15.5\left[\frac{L^*}{N_{Re}}\right]^{1/2} L^*(\rho^*+1) \quad (20)$$

$$C_2 = \left(\frac{18}{N_{Re}}\right)(\rho^*+1) \quad (21)$$

$$C_3 = \rho^*L^{*2}; \quad C_4 = \frac{0.029}{(N_{Re}L^{*3})^{1/2}\tau^3} \quad (22)$$

$$F^* = \frac{9\tau}{32}\left[\frac{1}{R^*}+\frac{1}{1-R^*}\right] \quad (23)$$

$$G^* = \frac{5\tau^3}{128}\left[\left(\frac{1}{R^*}\right)^2 - \left(\frac{1}{1-R^*}\right)^2\right] \quad (24)$$

$$H^* = \frac{(v_t/D_i\Omega_i)^2}{R^*-L^*} \quad (25)$$

$$V_t^* = 1 - R^* - W^* \quad (26)$$

The calculation of sphere trajectories depends on the simultaneous solution of the radial and tangential component Equations (18) and (19).

These equations can be analytically solved for a neutrally buoyant sphere which is not accelerating. For this case, the centripetal force will be zero (ρ^* is zero), and the derivatives d^2R^*/dT^{*2} and dV_t^*/dT^* will be zero. The result of algebraic manipulation and integration by separation of variables of the simplified forms of Equations (18) and (19) results in

$$W^* = -\frac{\tau^3}{2}\left[\frac{1-2R^*}{R^*(1-R^*)[32R^*(1-R^*)+9\tau]}\right] \quad (27)$$

$$T^* = C_4 \quad (4-9\tau)(8+9\tau)\ln\left[\frac{1-2R_i^*}{1-2R^*}\right] + 2[2(4-9\tau)+(8+9\tau)][(1-2R^*)^2 - (1-2R_i^*)^2] - 8[(1-2R^*)^4 - (1-2R_i^*)^4] \quad (28)$$

The graphical representations of this solution presented in Figures 2 and 3 demonstrate that the lag velocity changes sign at the midpoint between the cylinder walls. Thus, the equilibrium position of any sphere which is neutrally buoyant must be the midpoint of the annular region. This conclusion applies even to the case of a sphere which may experience acceleration. Note that in plotting ($R^* - 0.5$) against the scaled dimensionless time ($34.5\ N_{Re}^{1/2}\ L^{*3/2}\ \tau^3$) ($T^*$) in Figure 3, most of the dependence of the trajectory on τ is absorbed by the scaling for values of τ up to 0.30.

The assumption of zero acceleration in the tangential direction can be relaxed, and the density difference between spheres and liquid can be considered. It is, however, necessary to use numerical techniques to solve the resulting system of equations. The approximation of zero acceleration in the radial direction will be retained.

Consider approximating the acceleration in the tangential direction with the radial velocity. The validity of this approximation can be ascertained by considering the following equation:

$$\frac{dV_t^*}{dT^*} = \frac{d}{dT}(1-R^*-W^*) = -\frac{dR^*}{dT^*} - \frac{dW^*}{dT^*} \quad (29)$$

The assumption made in the numerical analysis is that the rate of change of the lag velocity is negligible.

The introduction of this assumption of zero acceleration in the radial direction, the approximation of the tangential velocity of the sphere by $(1 - R^*)$ in the centripetal force term of Equation (25), and the subsequent algebraic manipulation reduces the approximate problem to

$$W^* = \frac{-C_3H^* + C_2^2(1+2F^*)G^*}{C_1 - C_2^2(1+2F^*)(1+F^*)} \quad (30)$$

$$\frac{dR^*}{dT^*} = -C_2[W^*(1+F^*)+G^*] \quad (31)$$

Because the functions F^*, G^*, and H^* are functions of R^* only, the substitution of Equation (30) into (31) allows T^* to be expressed as the integral of a function of R^*, or

$$T^* = C_2\int_{R_i^*}^{R^*} \left[\frac{C_3H^* - C_2^2(1+2F^*)G^*}{C_1 - C_2^2(1+2F^*)(1+F^*)}(1+F^*) - G^*\right] dR^* \quad (32)$$

Numerical evaluation of the integral is easily carried out

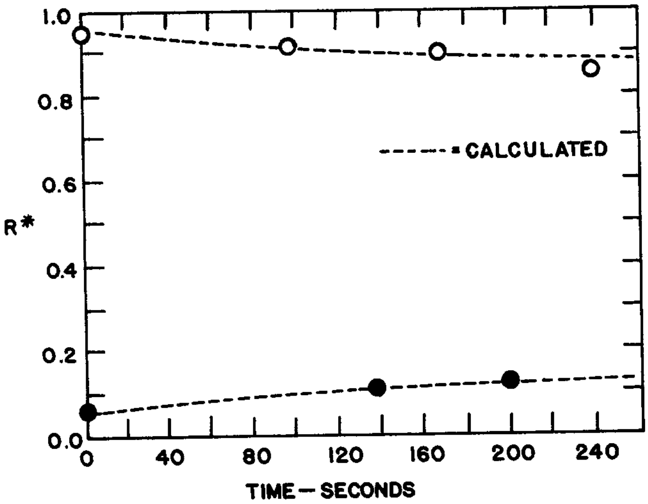

Fig. 5. Radial position vs. time: measured and calculated.

Experimental Conditions

Sphere diameter, mm.	1.699
Annular thickness, mm.	9.48
Rod angular velocity, sec.$^{-1}$	4.49
Liquid viscosity, poise	0.455
Liquid density, g./cc.	1.045
N_{Re} =	0.2990
τ =	0.1792
L^* =	5.380
ρ^* =	0.0044
Sphere diameter, mm.	0.592
Annular thickness, mm.	18.22
Rod angular velocity, sec.$^{-1}$	1.36
Liquid viscosity, poise	0.497
Liquid density, g./cc.	1.046
N_{Re} =	0.0085
τ =	0.0325
L^* =	2.800
ρ^* =	0.0010

once the constants C_1, C_2, and C_3 are specified.

COMPARISON OF SOME NUMERICAL SOLUTIONS WITH SOME EXPERIMENTALLY MEASURED TRAJECTORIES

A Fortran program for an IBM-7040 digital computer was written to evaluate the integral in (32) and thereby obtain radial trajectories for spheres. Simpson rule integrations performed by this program together with experimentally measured trajectories obtained by Halow and Wills (*10*) are presented in Figures 4 and 5. These figures represent the extremes of the migration phenomena studied. Intermediate results gave entirely comparable agreement between measurements and predictions (*11*). Only small discrepancies in the final equilibrium positions were found. These are thought to be due to a combination of experimental errors and the approximation of the Couette geometry with the infinite parallel plane system.

DISCUSSION

The model developed for the radial migration of spheres in Couette system allows trajectories for these spheres to be computed. It was found that experimentally measured migration occurred more rapidly than that predicted by the model unless the lift force predicted by Saffman was multiplied by a factor of 5; with this correction to the Saffman results, agreement between theory and experiment was obtained.

It is interesting to note that the factor of 5 has little effect on the shape of the trajectory curves except to shrink the time scale. The general shape of these curves is, in fact, determined by the lag velocity W^*, since this variable determines the relationship between lift force and radial position.

The fact that the wide range of conditions covered by the trajectory plots are all well fitted by the use of the single factor 5 indicates that the dependence of the migration phenomena on the system (viscosity, density, sphere, and Couette dimensions) is properly accounted for. If the functional dependence of the calculated trajectories on the system was incorrect, one could expect a definite variation in the value of the correction factor needed to give agreement between theory and experiment.

We have no explanation to offer at this time for the correction factor. It cannot be attributed to the approximations made for the drag forces, since if this were the source of the discrepancy, then the general shape of the trajectory curves would not be correct. The lag velocity W^* does directly depend on the drag terms.† The use of a steady state lift term (for what is an unsteady state phenomenon) also does not offer a proper explanation. Calculated trajectories including the additional lift terms given by Chang (*9*), which will account for the unsteady state nature of the problem, do not agree with measurements. Also, an inadequacy of this type would be expected to require a correction dependent on the system, not a single factor of 5.

Neglecting the acceleration terms d^2R^*/dT^{*2} and dW^*/dT^* introduces a small error for the conditions covered by the trajectory plots. The magnitudes of these terms were estimated for the experimental data and found to be small when compared with the other terms in Equations (18) and (19).

While the model presented above is approximate and semiempirical, it does accurately predict radial migration of spheres in Couette systems where the velocity profile can be considered linear. It is hoped that the experimental observations and mechanistic interpretation of the phenomena will be theoretically interesting and of practical use.

NOTATION

C_1, C_2, C_3 = collections of terms defined by Equations (20), (21), and (22)
D_s = sphere diameter
$\mathbf{e}_r, \mathbf{e}_t, \mathbf{e}_z$ = unit vectors associated with cylindrical coordinate system with origin at center of inner Couette cylinder
$\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ = unit vectors associated with Cartesian coordinate system with origin at center of sphere
$\mathbf{F}_b$ = body force on sphere
$\mathbf{F}_s$ = surface force on sphere
$\mathbf{f}^{(1)}$ = lift force on sphere in direction of $\mathbf{e}_r$
$\mathbf{f}^{(2)}$ = drag force on sphere in direction of $\mathbf{e}_r$
$\mathbf{f}_1^{(3)}$ = component of drag force on sphere in direction of $\mathbf{e}_t$
$\mathbf{f}^{(3)}$ = total drag force on sphere in direction of $\mathbf{e}_t$
F^* = collection of terms defined by Equation (23)

† We have compared the value of the drag acting on a nonlagging sphere with that calculated by the Faxen technique and found that to order $(r/R^*)^3$ it agrees to within 10%. We calculated this second result by merely superimposing the result of Wakiya (*15*) and that of Faxen (*16*). We have also found that the Wakiya and Faxen analysis can be used to prove that a sign change in the lag velocity must occur as predicted by Equation (27).

G^* = collection of terms defined by Equation (24)
g = acceleration of gravity
H^* = collection of terms defined by Equation (25)
L^* = dimensionless radius ratio = $R_i/(R_o - R_i)$
m_s = mass of sphere
N_{Re} = Reynolds No. = $D_s^2\Omega_i\rho_1/\mu$
$\mathbf{n}$ = unit vector normal to surface of sphere
$\mathbf{q}_1$ = velocity field reflected from inner wall
$\mathbf{q}_2$ = velocity field reflected from outer wall
r = radial coordinate of cylindrical coordinate system associated with Couette system
r_s = radial coordinate of sphere in cylindrical coordinate system
r_s^1 = radial coordinate of spherical coordinate system with origin at center of sphere
R_i = radius of inner Couette cylinder
R_o = radius of outer Couette cylinder
R^* = $(r - R_i)/R_o - R_i)$ = dimensionless radial position
t = time
T^* = $t\Omega_i$ = dimensionless time
$\mathbf{u}$ = fluid velocity
u_t = tangential velocity of fluid in Couette system
$\mathbf{u}_{fs}$ = free stream velocity
$\mathbf{v}$ = velocity of center of sphere
v_r = velocity of sphere in $\mathbf{e}_r$ direction
v_t = velocity of sphere in $\mathbf{e}_t$ direction
V^* = $v/(R_i\Omega_i)$ = dimensionless velocity
W^* = $w/(R_i\Omega_i)$ = dimensionless lag velocity
w = lag velocity
x, y, z = Cartesian coordinates with origin at center of sphere
z_s = vertical coordinate of sphere in Couette region

Greek Letters

$\mathfrak{S}$ = stress tensor
κ = gradient in free stream velocity
μ = viscosity
ρ_s = density of sphere
ρ_1 = density of liquid
ρ^* = reduced density difference = $(\rho_1 - \rho_s)/\rho_s$
τ = ratio sphere diameter to annular thickness of Couette region = $D_s/(R_o - R_i)$
ω_s = tangential velocity of sphere surface due to rotation about its own axis
Ω_s = angular speed of rotation of sphere about its own center
Ω_i = angular speed of rotation of inner Couette cylinder

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Absorption and Scattering of Thermal Radiation by a Cloud of Small Particles

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An experimental investigation was made of the absorption and scattering of thermal radiation by a cloud of small, spherical, micronsized, aluminum oxide particles in a plane-parallel enclosure. By using carbon tetrachloride and carbon disulphide as suspending media, transmission measurements were made with collimated and diffuse sources for wavelengths from 2 to 11μ. By employing a two-flux diffuse model, the data were correlated to obtain absorption and backscattering cross sections. From the diffuse and collimated data, a backscatttering coefficient was found which varied linearly with relative refractive index. With the developed coefficients, the absorptivities of the cloud of particles were determined.

For clouds of small particles with diameters of 10μ magnitude, the prediction of heat transfer rates by radiant transfer to and from the particles is not only mathematically complex but also subject to uncertainties caused by nonspherical shape, size distribution, and nonuniform

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